21-12-17

10022

Total Pages : 3

PC-1225/NH

BS/2117

ANALYSIS–I

Paper–II

(Semester–III)

Time : Three Hours] [Maximum Marks : 36

Note : Attempt *two* questions each from Section A and B carrying 5.5 marks each, and the entire Section C consisting of 10 short answer type questions carrying 14 marks in all.

SECTION–A

I. State and prove Fundamental theorem of Integral calculus. 5½

II. Prove that every continuous function is Riemann integrable. 5½

III. Let $f(x) = 3x + 1$ on $[1, 2]$. Prove that f is R-integrable on $[1, 2]$ and $\int_1^2 f(x)\,dx = \frac{11}{2}$. 5½

IV. By considering the integral $\int_{n}^{n+1} \frac{1}{x} dx$; $n > o$ prove that

$$\frac{1}{n+1} \le \log\left(1+\frac{1}{n}\right) \le \frac{1}{n}.$$ 5½

SECTION–B

V. Find the directional derivative of $f(x, y, z) = x^2y^2z^2$ at the point (1, 1, –1) in the direction of the tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$. 5½

VI. Verify Green's theorem in the plane for

$$\oint_C \left[(xy + y^2)\, dx + x^2 dy\right]$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5½

VII. The necessary and sufficient condition for the vector function $\overrightarrow{f(t)}$ to have constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = \vec{0}$.

5½

VIII. Prove the following :

(a) div grad $v = \nabla^2 v$.

(b) curl grad $v = \vec{0}$

(c) div curl $\vec{v} = 0$ 5½

SECTION–C

IX. Attempt all the following :

(a) State Second mean value theorem of Integral calculus. 1

(b) Define Gradient of a scalar point function. 1

(c) Define Solenoidal vector. 1

(d) Determine the constant a so that the vector $\vec{F} = (x + 3y)\,\hat{i} + (y - 2z)\,\hat{j} + (x + az)\,\hat{k}$ is solenoidal. 2

(e) State First mean value theorem of Integral calculus. 1

(f) State Divergence theorem. 2

(g) Define Constant vector with an example. 2

(h) Define Acceleration. 1

(i) Evaluate $\int_{\partial}^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx.$ 2

(j) Define Lower Riemann Sum. 1

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STATICS

Paper–III

Semester–III

Time Allowed : 3 Hours]   [Maximum Marks : 36

Note : The candidates are required to attempt **two** questions each from Sections A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

SECTION—A

1. (a) Find the magnitude and direction of the resultant of two forces acting at a point at an angle α. 3

(b) If a force be resolved into two components, one of which is at right angles to the force and equal to it in the magnitude, find the direction and magnitude of the other. 2½

2. Forces of 1, 3, 5, 7, 9 and 11 newtons act along the sides AB, BC, CD, DE, EF, FA respectively of a regular hexagon ABCDEF, taken in order. Find the moment of each force about A and their resultant. 5½

3. (a) State and prove Generalized theorem of resolved parts. 3

(b) ABC is a triangle and O a point in its plane. A force R acts along AO. Resolve R into two forces parallel to it and acting at B and C respectively where O is the circumcentre of the triangle. 2½

4. (a) Prove that two Coplanar couples of equal moments and in the opposite sense balance each other. 3

(b) Forces P, 2P, 3P, 4P act along the sides AB, BC, CD, AD of a square. Reduce the system to a force at A and a force along BC. 2½

SECTION—B

5. Two weights P and Q are suspended from a fixed point O by strings OA, OB which are kept apart by a light rod AB. If the string makes angles α and β with the rod, show that the angle θ which the rods makes with the vertical is given by

$$\tan\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}.$$ 5½

6. (a) If three non-collinear forces acting at a point are in equilibrium, then each is proportional to the sine of the angle between the other two. 3

(b) ABCD is a parallelogram and P is any point in its plane. A particle at P is attracted towards A and C by forces proportional to PA and PC respectively and repelled from B and D by forces proportional to PB and PD. Show that P is in equilibrium in all positions. The constant of proportionality being same in both the cases. 5

7. (a) Find the centre of gravity of a curved surface of a Hollow cone. 3

(b) A rod of weight 10 kg. is supported by a hinge at A and carries a weight of 20 kg at B. It is tied back so as to make an angle of 60° with the upward vertical by a horizontal string attached at B. Find the tension of the string and the reaction at A. 2½

8. (a) Two uniform rods AB, BC of lengths 2a, 2b respectively are rigidly united at B and are suspended freely from A. If they rest inclined

at angles θ, φ respectively to the vertical show

that $\dfrac{\sin\theta}{\sin\varphi} = \dfrac{b^2}{a(a+2b)}$. 3

(b) A body is placed on a rough plane inclined to the horizon at an angle greater than angle of friction and is supported by force acting at an angle θ with the inclined plane. Find the limits between the force must lie. 2½

SECTION—C

9. Answer the following questions : 7×2=14

(i) At what angle must force of 8 kg and 3 kg be inclined in order that their resultant may be 7 kg ?

(ii) Prove that any system of Coplanar forces acting on a rigid body can be reduced to a single force or a single couple.

(iii) State $\lambda - \mu$ theorem.

(iv) Find the necessary and sufficient conditions of equilibrium of a number of coplanar-concurrent forces.

(v) Find centre of gravity of a tetrahedron.

(vi) Determine the magnitude, direction of the resultant of any number of Coplanar forces.

(vii) State laws of friction.

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ADVANCE CALCULUS–I

(Semester–III)

Time : Three Hours] [Maximum Marks : 36

Note : Attempt *two* questions each from Section A and B carrying 5.5 marks each, and the entire Section C consisting of 07 short answer type questions carrying 2 marks each.

SECTION–A

I. Let $u_1 = \frac{x_1}{x_5}, u_2 = \frac{x_2}{x_5}, u_3 = \frac{x_3}{x_5}, u_4 = \frac{x_4}{x_5}$ and

$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 1$. Then show that

$$\frac{\partial(u_1, u_2, u_3, u_4)}{\partial(x_1, x_2, x_3, x_4)} = \frac{1}{x_5^6}.$$ 5½

II. By using Taylor's theorem, expand $x^4 + x^2y^2 - y^4$ about (1, 1) upto the terms of second degree. 5½

III. State and prove Schwarz's theorem. 5½

IV. Find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a^2$. 5½

SECTION–B

V. Evaluate by changing the order of integration of

$$\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy.$$ 5½

VI. Show that $\iiint_{x^2+y^2+z^2 \le 1} (x^2 + y^2 + z^2) \, dx \, dy \, dz = \frac{4\pi}{5}$.

5½

VII. Find the centroid of an ellipse with mass density one.

5½

VIII. Find the volume of a cylinder with base radius R and height H. 5½

SECTION–C

IX. (a) Use definition to show that

$$\lim_{(x, y) \to (2, 1)} (3x - 4y) = 2.$$

(b) If $H = f(y - z, z - x, x - y)$ Prove that

$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0.$$

(c) Find the area of the circle using the double integration.

(d) Show that the function $f(x)$ = sin x + sin y is differentiable at every point of R^2.

(e) Find the moment of inertia of a square region of unit density about one of its side, the side being $2a$.

(f) Evaluate $J_f(x, y)$ for $f(x, y) = (x + y, (x + y)^2)$.

(g) If $z = f\left(\frac{y}{x}\right)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$. (2×7=14)

$\frac{1}{M} \iint x\mu(x,y)\, dx\, dy$